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Use a Riemann sum to evaluate the definite integral $\int_a^b f(x) dx$

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$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k^*) \Delta x$$

We can always use these formulas to find Δx and x_k^*

$$\Delta x = \frac{b-a}{n} \quad x_k^* = a + (k-1)\Delta x$$

Substitute this back in to the Riemann sum:

$$\begin{aligned} \int_a^b f(x) dx &= \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k^*) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(a + (k-1)\Delta x\right) \frac{b-a}{n} \\ &= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=1}^n f\left(a + (k-1)\Delta x\right) \\ &= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=1}^n f\left(a + (k-1)\frac{b-a}{n}\right) \end{aligned}$$

Use the summation formulas:

$$\begin{aligned} \int_a^b f(x) dx &= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=1}^n f\left(a + (k-1)\frac{b-a}{n}\right) \\ &= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=1}^n \left[c + d\left(a + (k-1)\frac{b-a}{n}\right) \right] \\ &= \lim_{n \rightarrow \infty} \frac{b-a}{n} \left[\sum_{k=1}^n c + d \sum_{k=1}^n \left(a + (k-1)\frac{b-a}{n}\right) \right] \\ &= \lim_{n \rightarrow \infty} \frac{b-a}{n} \left[nc + d \left(na + \frac{(n-1)n}{2} \frac{b-a}{n} \right) \right] \end{aligned}$$

We can verify by using the Fundamental Theorem of Calculus:

$$\begin{aligned} \int_a^b f(x) dx &= F(b) - F(a) \\ &= \left[c x + d \left(a x + \frac{x^2}{2} \frac{b-a}{n} \right) \right] \Big|_a^b \\ &= \left[c b + d \left(a b + \frac{b^2}{2} \frac{b-a}{n} \right) \right] - \left[c a + d \left(a^2 + \frac{a^2}{2} \frac{b-a}{n} \right) \right] \end{aligned}$$

